Understanding Feedforward Models for FRC

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1 Introduction

The most important part of a well-tuned control loop is the feedforward model. The idea of feedforward is to take into account what we know about the system to produce a better (faster rise, less overshoot, less error) response than pure feedback. This is because a well designed model makes the output of the system look to the feedback loop how it's expected to- having a linear relationship between the input and the process variable (the output we are trying to control). Without feedforward, most systems will end up being sluggish, with high overshoot and long settling times. Overall, adding feedforward also makes the system easier to tune. Feedforward is of course used in conjunction with "feedback," (the P, I, and D terms) which should in principle only work to counter non-ideal properties like friction, motor heating, air resistance, etc. Calculating feedforward requires knowing information about the system, using the same sensors as we use for feedback.

2 A Motor Primer

Generally, we control motors by feeding them a certain amount of voltage in the range -12 to 12 Volts. Under no load, this causes them to spin faster the more voltage is applied. We can write this relationship as

$$\omega = K_v V \tag{1}$$

, where ω is the rotational speed of the motor, V is the applied voltage, and K_v is the motor speed constant, a property of the motor type used.

Also interesting, even more so than speed, is the amount of torque produced by the motor. This dynamic is also fairly simple, $\tau = K_t I$, where τ is the torque, I is the current drawn by the motor, and K_t is the motor torque constant.

Since we generally control voltage and not current, we should consider how the torque dynamic relates to voltage directly. In general, we will only care about torque at stall. At stall, by Ohm's Law, $I = \frac{V}{R}$. Thus, at stall,

$$\tau = K_t \frac{V}{R} \tag{2}$$

As motor resistance is constant at stall, torque at stall is proportional to applied voltage.

The full DC motor voltage balance equation is slightly more complicated than these two pieces, but they will be sufficient for our purposes of developing single-order speed and position models. Note that the constants are exact reciprocals in the correct SI units.

2.1 Determining K_t and K_v

From the equation $\omega = K_v V$, determining K_v is as simple as plugging in known values for ω and V, (such as 18730 RPM and 12 V for a 775pro) and solving. If multiple equivalent motors are geared together, then their effective K_v is the same as one motor. Any gear reductions applied decrease K_v by a proportional amount.

From $\tau = K_t I$, the same process can be applied using the known stall torque and stall current of the motor. If multiple equivalent motors are geared together, the effective K_t is multiplied by the number of motors. Any gear reductions applied increase K_t by a proportional amount.

3 Derivation of Simple Feedforward Models

3.1 The Flywheel

The simplest feedforward model, and the one that most are introduced to as *the* definition of feedforward, is that of a speed constant times the setpoint, in order to set a motor to spin at that speed. To derive this, consider the speed dynamic $\omega = K_v V$. Knowing the motor K_v and any gear reductions G (applied as a division on the right side) allows one to solve for the desired voltage as a function of the desired speed.

$$V = \frac{G\omega}{K_v} \tag{3}$$

Example Consider a 775pro motor with $K_v = 1561\,\frac{\text{RPM}}{\text{V}}$ and a gear reduction of 3:1. Thus the feedforward function is $V = \frac{3\omega}{1561} = 0.0019\omega$

3.2 Tank Drivetrain

FRC drivetrains are a very different beast than simple flywheels, although they are objectively the same. The difference is that drivetrains have a much higher amount of static friction and effective moment of inertia (resistance to movement, in an angular sense) that can't be ignored.

To consider these effects, we use a 3-term feedforward, taking into account static voltage to initially move wheels (C_s), voltage to keep moving at a velocity V (C_v), and voltage required to accelerate at A (C_a).

$$V_{ff}(V, A) = C_s + C_v V + C_a A$$

Theoretically calculating these constants is beyond the scope of this paper or impossible, as this model considers non-ideal behavior that is however too significant to ignore. However, it is fairly simple to empirically determine them. The essence of the determination is collecting data points at varying velocities and accelerations and determining the best-fit plane to these points as a function of actual applied voltage to the motors.

A more in-depth discussion of this model, as well as the process of determining the constants, can be found in this paper by team 449.

3.3 The Wrist/Arm

A common application of control loops, seen in almost every game, is to control the angle of a wrist or arm. The major force opposing motion of such an arm is gravity, which effects a different torque on the arm based on its position through the rotation. Our goal is to find the moment arm,

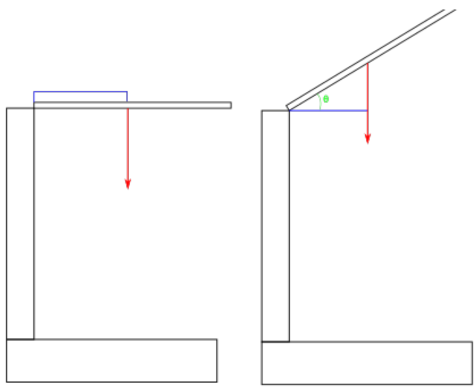

Figure 1: Two positions and forces of a wrist or arm mechanism

which is the distance between the joint and the vector of the force of gravity. Let S (blue) be the distance from the joint to the center of mass of the arm.[1] Let also the angle θ be the angle of the arm above the horizontal. To find the length of the moment arm L (blue), recognize that L, S, and θ create a triangle such that $\cos\theta = \frac{L}{S}$. Thus, the moment arm is $L = S\cos\theta$.

To find the torque applied by gravity, simply multiply the weight of the arm W by the length of the moment arm.

$$\tau_g = WS\cos\theta$$

Note that if the wrist or arm is counterbalanced by a torsion spring, this must be considered when designing a feedforward.

To find the required voltage to hold the arm at a position θ, simply set the torque from gravity equal to the torque provided by the motor (2) and solve.

$$\begin{aligned} \tau &= \tau_g \\ K_t \frac{V}{R} &= WS\cos\theta \\ V &= \frac{WSR}{K_t}\cos\theta \end{aligned} \tag{4}$$

As a check for understanding, consider two positions of the arm and compare to intuition. When the arm is perfectly vertical ($\cos 90° = 0$), no torque will be required. When the arm is horizontal ($\cos 0° = 1$), the required torque will be maximized.

Example Consider a CIM motor with a K_t of $0.163\,\frac{\text{in lb}}{\text{A}}$ and an internal resistance of $0.09\,\Omega$. This motor will control a wrist with a weight of 12 lbs, centered 10 inches from the pivot. The CIM feeds into a 50:1 planetary gearbox, increasing the effective K_t to $8.15\,\frac{\text{in lb}}{\text{A}}$. Thus we find the feedforward function to be

$$V(\theta) = \frac{12\,\text{lb}\cdot 10\,\text{in}\cdot 0.09\,\Omega}{8.15\,\frac{\text{in lb}}{\text{A}}}\cos\theta = 1.33\,\text{V}\cdot\cos\theta$$

3.4 The Elevator

Another common mechanism is the elevator, with multiple stages arranged in cascade or continuous rigging, driven by a spool or chain. All are effectively variants of lifting weights with a spool.

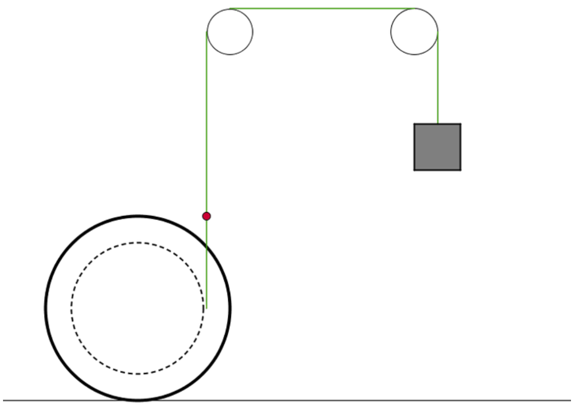

Figure 2: Weight on a spool. From https://arcsecond.wordpress.com

Consider a spool of radius R with a weight of W hanging from a weightless string attached too and wrapped around the spool. The torque on the spool is simply $\tau_g = RW$. Note that this

[1]Determining the center of mass can be non-trivial- one could use CAD, or set up a rig to balance the arm at various lengths.

torque does not vary with any function of the elevator, and can be considered a static addition to the output.

Determining the effective weight is not always trivial. With a continuous elevator, the effective weight is the sum of the weight of the engaged stages. However, with cascade rigging, each stage's effective weight increases with the number of stages. That is, the first stage has a 1x effective weight, the second has an effective weight of 2x its actual weight, and the third stage has a 3x effective weight multiplier. However, this is balanced out by being able to use constant-force springs, which subtract 1x their force from the effective weight of the mechanism.

When chain rigging is used, the pitch diameter of the driving sprocket is used as the spool radius.

As noted earlier, the effective weight of a continuous elevator increases with the number of stages engaged

At this point, the derivation of the feedforward voltage is much the same as with the wrist/arm.

$$\begin{aligned} \tau &= \tau_g \\ K_t \frac{V}{R} &= WR \\ V &= \frac{WR}{K_t} \end{aligned} \tag{5}$$

Example Consider two 775pro motors geared 21:1 with a 0.5" diameter spool. These motors individually have a K_t of $0.521\,\frac{\text{in lb}}{\text{V}}$ and the effective K_t of the gearbox is thus $0.521 * 2 * 21 = 21.82\,\frac{\text{in lb}}{\text{V}}$. The motors are controlling a cascade elevator where the first stage weighs 10 lbs and the second stage (carriage) weighs 20 lbs. Therefore the effective weight will be $10 + 20 * 2 = 50$ lbs.

Thus we find the feedforward function to be

$$V_{static} = \frac{50\,\text{lb} \cdot 0.5\,\text{in}}{21.82\,\frac{\text{in lb}}{\text{V}}} = 1.15\,\text{V}$$

4 Conclusion and Further Reading

This has been a brief overview of the derivation of several feedforward models. The actual content, math, and derivation is not quite as important as understanding the concept and purpose of modeling the system, as well as understanding the physics used to derive the models. Although modeling can become much more complex, covering multiple orders of the system, or considering how different inputs and outputs interact, these simple single-input-single-output single state models serve the purpose of FRC quite well.

For further reading on DC motor theory, Brushed DC Motor Theory from Northwestern is recommended. This page covers the physics behind brushed DC motors, as well as the resulting equations.

For more advanced control theory, "Practical Guide to State-space Control" is an accessible introduction to modern control, as well as prerequisite and following topics. It also includes more advanced versions of some of the models derived in this paper.